Ageing properties in off-equilibrium critical relaxation of 3D diluted Ising ferromagnets

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The non-equilibrium relaxation of magnetic systems close to critical point demonstrates a wide range of interesting phenomena such as critical slowing down, ageing properties, and violation of the fluctuation-dissipation theorem (FDT). According to dynamical scaling the relaxation time $t_{rel} \to |T - T_c|^{-z\nu}$ and magnetic system does not achieve an equilibrium state at critical point. During out-of-equilibrium stage of dynamics for $t \ll t_{rel}$ ageing phenomena occur with two-time dependence of correlation and response functions on characteristic time variables as waiting time t_w and time of observation $t - t_w$ with $t > t_w$ [1]. It was shown that the time correlation function decays more slowly with increasing waiting time t_w.

The relationship between time correlation function $C(t, t_w)$ and response function $R(t, t_w)$ can be written as

$$R(t, t_w) = \frac{X(t, t_w)}{T} \frac{\partial C(t, t_w)}{\partial t_w}, \tag{1}$$

where $X(t, t_w)$ is so called fluctuation-dissipation ratio (FDR). FDT states that $X(t, t_w) = X(t - t_w) = 1$ in equilibrium.

Using Monte-Carlo simulations we have investigated ageing properties of three-dimensional Ising model with point-like nonmagnetic impurities. The spin concentrations p were taken as equal to $p = 1, 0,8$ and $0,6$. The investigations were carried out on cubic lattices. We used a high-temperature initial state with small initial magnetization $m_0 \ll 1$. Analysis of autocorrelation function behavior showed the realization of ageing in systems during out-of-equilibrium stage of dynamics for each spin concentration.

For checking violation of FDT we have used two ways to find fluctuation-dissipation ratio. Firstly, using Metropolis dynamics, we simulated Ising model in the presence of small bimodal random magnetic field h on lattice after t_w with distribution $< h >= 0$ [2]. In this way FDR can be calculated using integrated susceptibility $\chi(t, t_w)$ and final expression is

$$X(t, t_w) = -\lim_{C \to 0} \frac{\partial (T\chi(t, t_w))}{\partial C(t, t_w)}. \tag{2}$$

As result of investigations we obtained the following values of FDR: $X^\infty(p = 1) = 0,391(12)$, $X^\infty(p = 0,8) = 0,419(11)$ and $X^\infty(p = 0,6) = 0,443(6)$.

Another way of FDR definition is to calculate the response function $R(t, t_w)$ and derivative of correlation function without the use of the random magnetic field h. In this case FDR can be estimated through calculation of some complicated correlation functions as [3]

$$X(t, t_w) = TR(t, t_w)/(\frac{\partial C(t, t_w)}{\partial t}) = \frac{\sum_{i=1}^{N} < \sigma_i \left[\sigma_k(t_w + 1) - \sigma_k^{Weiss}\right] >}{\sum_{i=1}^{N} < \sigma_i(t)(\sigma_i(t_w + 1) - \sigma_i(t_w)) >}. \tag{3}$$

The sum in this expression includes all sites at the lattice, σ_i is spin value at site i. $\sigma_i^{Weiss} = tanh(1/T \sum_{<j \neq i>} \sigma_j)$ (where sum pass through neighboring spins around site i). The obtained final values of FDR are $X^\infty(p = 1) = 0,381(16)$, $X^\infty(p = 0,8) = 0,426(10)$ and $X^\infty(p = 0,6) = 0,451(10)$. Thereby, it was proved the violation of FDT in disordered three-dimensional Ising model and was shown that the non-equilibrium critical dynamics of this model is characterized by new universal FDR with $X^\infty_{disorder}(p < 1) > X^\infty_{pure}(p = 1)$.

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